Kwantowa teoria gier w podejmowaniu decyzji

Marek Szopa
Katedra Badań Operacyjnych
marek.szopa@uekat.pl

University of Economics in Katowice

Motywacje

- Optymalizacja decyzji podejmowanych w warunkach konkurencji
- Zabezpieczenie przed manipulacją wynikiem gry
- Poszukiwanie nowego typu strategii graczy, innych niż strategie czyste, mieszane czy skorelowane rozkłady prawdopodobieństwa
- Demitologizacja gier kwantowych

Plan

- Optymalizacja wyników przez równowagi skorelowane Aumanna
- Gry kwantowe w schemacie EWL
- Pareto-optymalność kwantowych strategii mieszanych
- Symulacje IBM Q gry kwantowej „bezmyślny kierowca”

Games and probability distributions

We consider two player *games*

$$G = \left(N, \left\{S_X\right\}_{X \epsilon N}, \left\{P_X\right\}_{X \epsilon N}\right)$$

where:

$N = \{A, B\}$ is the set of players

$S_A = \{A_0, A_1\}$, $S_B = \{B_0, B_1\}$ are possible pure strategies

$P_X: S_A \times S_B \to \left\{v_{ij}^X \epsilon \mathbb{R} \,\middle|\, i,j = 0,1\right\}, X = A, B$, are payoff functions, represented by the game bimatrix

$$\begin{pmatrix} (v_{00}^A, v_{00}^B) & (v_{01}^A, v_{01}^B) \\ (v_{10}^A, v_{10}^B) & (v_{11}^A, v_{11}^B) \end{pmatrix}$$

Let

$$\Delta(S_A \times S_B) = \left\{\sum_{i,j=0,1} \sigma_{ij} A_i B_j \,\middle|\, \sigma_{ij} \geq 0, \sum_{i,j=0,1} \sigma_{ij} = 1\right\}$$

be the set of *probability distributions* over $S_A \times S_B$

Correlated equilibria

Probability distribution $\{\sigma_{ij}\}_{i,j=0,1}$ over set of strategies $(A_i, B_j)_{i,j=0,1}$ of the game G is a *correlated equilibrium* iff

$$\sum_{j=0,1} \sigma_{ij}\, v_{ij}^{A} \geq \sum_{j=0,1} \sigma_{ij}\, v_{-ij}^{A} \quad \text{and} \quad \sum_{j=0,1} \sigma_{ji}\, v_{ji}^{B} \geq \sum_{j=0,1} \sigma_{ji}\, v_{j(-i)}^{B}$$

where $-i \neq i$ is the index of the remaining strategy.

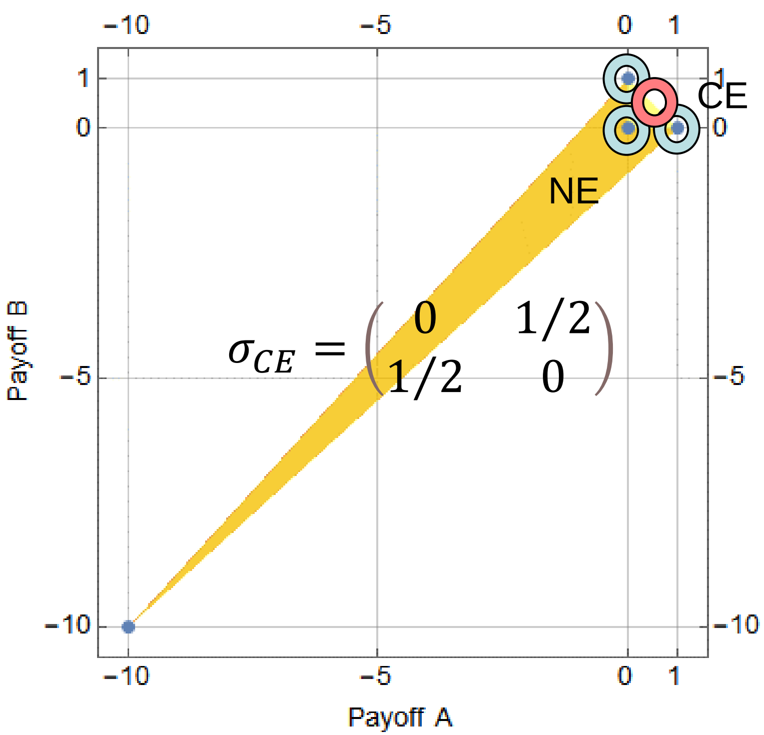
Efficiency of selected classical games

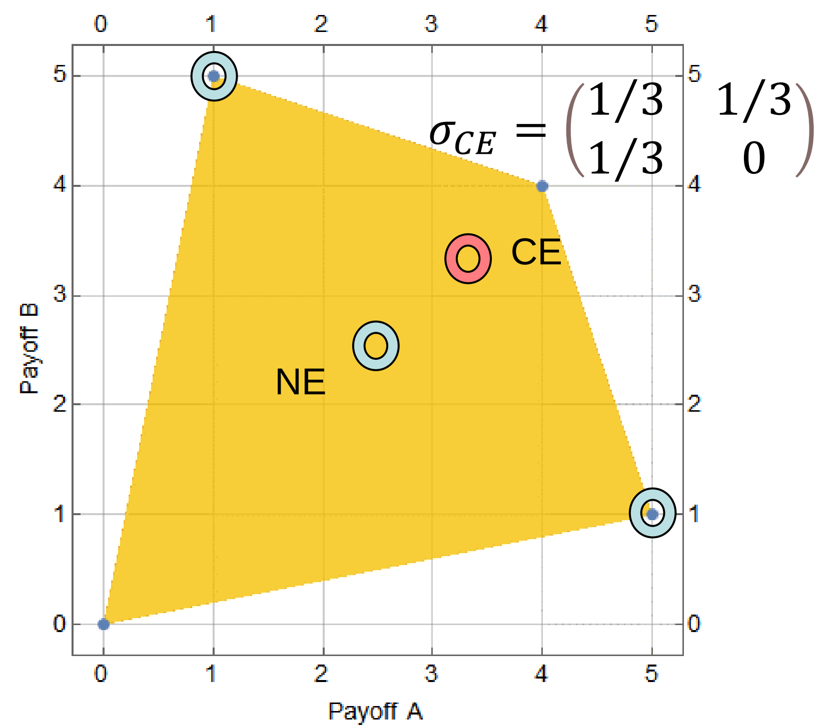
chicken		Driver B	
Driver A		B_0	B_1
	A_0	$(0,0)$	$(0,1)$
	A_1	$(1,0)$	$(-10,-10)$

chicken 2		Player B	
Player A		B_0	B_1
	A_0	$(4,4)$	$(1,5)$
	A_1	$(5,1)$	$(0,0)$

$$\sigma_{00} \le 10\sigma_{01}, \sigma_{00} \le 10\sigma_{10}$$
$$10\sigma_{11} \le \sigma_{01}, 10\sigma_{11} \le \sigma_{10}$$

$$\sigma_{00} \le \sigma_{01}, \sigma_{00} \le \sigma_{10}$$
$$\sigma_{11} \le \sigma_{01}, \sigma_{11} \le \sigma_{10}$$





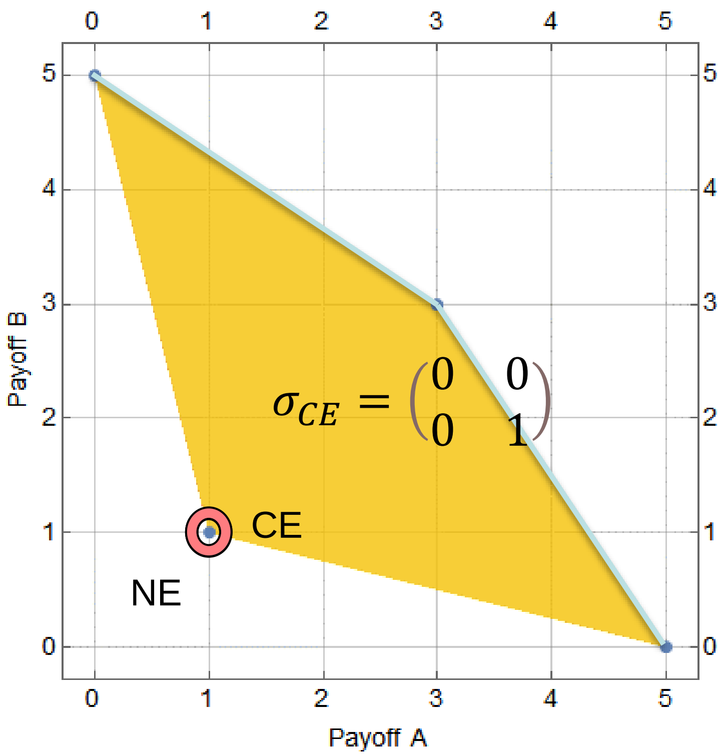
Efficiency of selected classical games

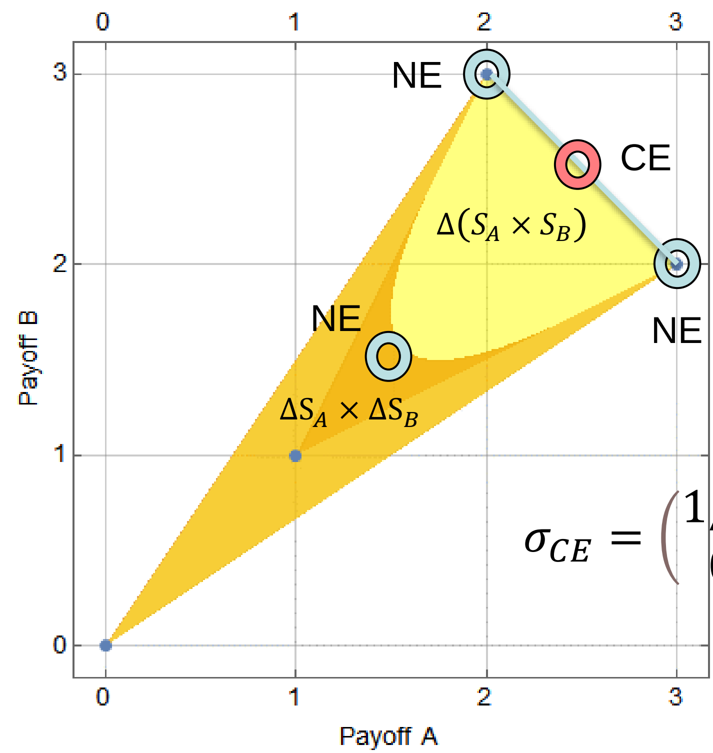
prisoner's dilemma		Bob	
		B_0	B_1
Alice	A_0	$(3,3)$	$(0,5)$
	A_1	$(5,0)$	$(1,1)$

battle of the sexes		Bob	
		B_0	B_1
Alice	A_0	$(3,2)$	$(1,1)$
	A_1	$(0,0)$	$(2,3)$

$$\sigma_{00} = \sigma_{01} = \sigma_{10} = 0$$
$$\sigma_{11} = 1$$

$$3\sigma_{00} \geq \sigma_{01}, \sigma_{00} \geq 3\sigma_{10}$$
$$3\sigma_{11} \geq \sigma_{01}, \sigma_{11} \geq 3\sigma_{10}$$

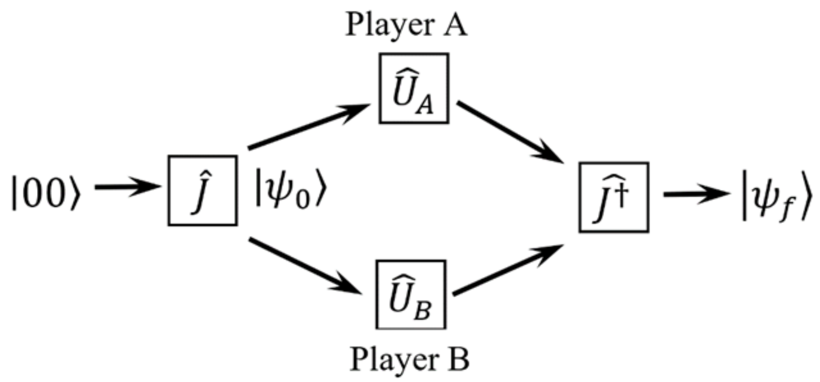




$$\sigma_{CE} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

EWL Quantum Game

The Eisert-Wilkens-Lewenstein quantum game is based on the scheme:



where: $|00\rangle$ is the initial state

$\hat{J} = \frac{1}{\sqrt{2}}(\hat{I} + i\sigma_x \otimes \sigma_x)$, $J^\dagger$ are the entangling, disentangling operators,

$$\hat{U}_X(\theta_X, \alpha_X, \beta_X) = \begin{pmatrix} e^{i\alpha_X}\cos\frac{\theta_X}{2} & ie^{i\beta_X}\sin\frac{\theta_X}{2} \\ ie^{-i\beta_X}\sin\frac{\theta_X}{2} & e^{-i\alpha_X}\cos\frac{\theta_X}{2} \end{pmatrix}, X = A, B,$$

$\left|\psi_f\right\rangle = \sum_{i,j=0,1} p_{ij}\ |ij\rangle$, is the final state defining the game payoffs

Quantum game payoffs

$\Pi_X: SU(2) \times SU(2) \to \mathbb{R}$ are payoff functions defined by:

$$\Pi_X\left(\hat{U}_A, \hat{U}_B, \gamma\right) = \sum_{k,l=0}^{1} v_{k,l}^X \left|\left\langle \Psi_{k,l}(\gamma)\right| U_A \otimes U_B \left|\Psi(\gamma)\right\rangle\right|^2, \; X = A, B$$

$$|\Psi_{k,l}(\gamma)\rangle = C_k \otimes C_l |\Psi(\gamma)\rangle$$

In case of a fully quantum case $\gamma = \pi/2$:

$$\Pi_X(\hat{U}_A, \hat{U}_B) = \sum_{k,l=0,1} |p_{kl}|^2 v_{kl}^X, \quad X = A, B,$$

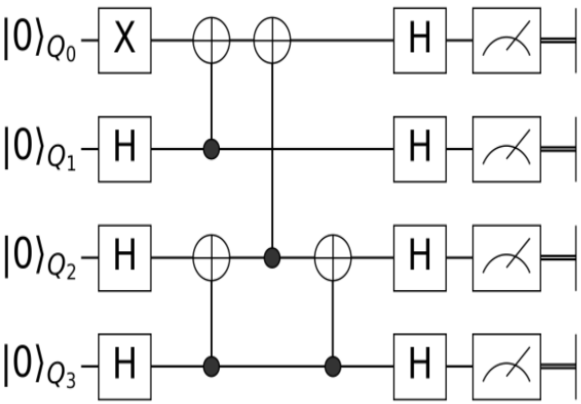
where:

$$|p_{00}|^2 = \cos\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\cos(\alpha_A + \alpha_B) + \sin\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\sin(\beta_A + \beta_B),$$

$$|p_{01}|^2 = \cos\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\cos(\alpha_A - \beta_B) + \sin\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\sin(\alpha_B - \beta_A),$$

$$|p_{10}|^2 = \cos\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\sin(\alpha_A - \beta_B) + \sin\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\cos(\alpha_B - \beta_A),$$

$$|p_{11}|^2 = \cos\frac{\theta_A}{2}\cos\frac{\theta_B}{2}\sin(\alpha_A + \alpha_B) - \sin\frac{\theta_A}{2}\sin\frac{\theta_B}{2}\cos(\beta_A + \beta_B).$$



EWL with Frąckiewicz-Pykacz parameterization

Let us restrict the set of quantum strategies to

$$\widehat{U}_X(\theta_X, \phi_X) = \begin{pmatrix} e^{-i\phi_X}\cos\frac{\theta_X}{2} & -e^{-i\phi_X}\sin\frac{\theta_X}{2} \\ e^{i\phi_X}\sin\frac{\theta_X}{2} & e^{i\phi_X}\cos\frac{\theta_X}{2} \end{pmatrix}$$

$$\widehat{P_0} = \widehat{U}(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\widehat{P_x} = \widehat{U}\left(\pi, \frac{3\pi}{2}\right) = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix},$$

$$\widehat{P_y} = \widehat{U}(\pi, 0) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},$$

$$\widehat{P_z} = \widehat{U}\left(0, \frac{3\pi}{2}\right) = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}.$$

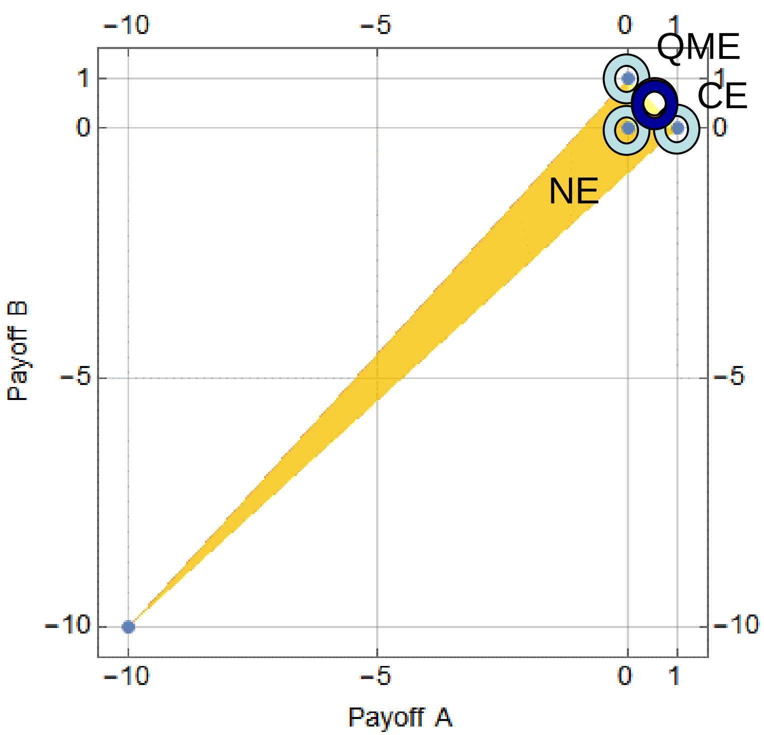
- In this parameterization, there are additional Nash equilibria in pure strategies
- F-P parametrization is invariant with respect to strongly isomorphic transformation of input games

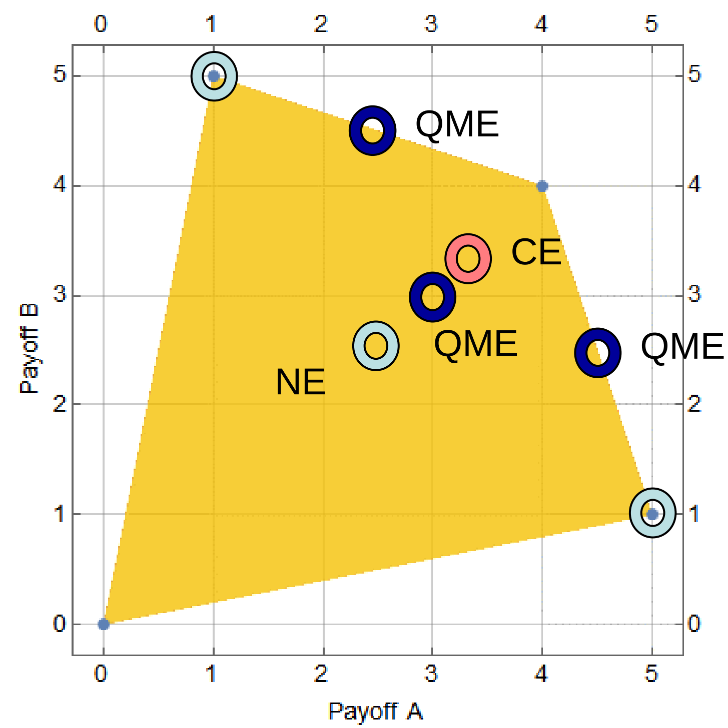
Quantum Mediated Equilibria

		Player B			
		$\widehat{P_0}$	$\widehat{P_x}$	$\widehat{P_y}$	$\widehat{P_z}$
Player A	$\widehat{P_0}$	(v^A_{00}, v^B_{00})	(v^A_{01}, v^B_{01})	(v^A_{10}, v^B_{10})	(v^A_{11}, v^B_{11})
	$\widehat{P_x}$	(v^A_{10}, v^B_{10})	(v^A_{11}, v^B_{11})	(v^A_{00}, v^B_{00})	(v^A_{01}, v^B_{01})
	$\widehat{P_y}$	(v^A_{01}, v^B_{01})	(v^A_{00}, v^B_{00})	(v^A_{11}, v^B_{11})	(v^A_{10}, v^B_{10})
	$\widehat{P_z}$	(v^A_{11}, v^B_{11})	(v^A_{10}, v^B_{10})	(v^A_{01}, v^B_{01})	(v^A_{00}, v^B_{00})

$\sigma^A = \left(\frac{1}{2}, 0, 0, \frac{1}{2}\right), \sigma^B = (0, \frac{1}{2}, \frac{1}{2}, 0)$

$\sigma^A = \left(\frac{1}{2}, 0, \frac{1}{2}, 0\right), \sigma^B = (\frac{1}{2}, \frac{1}{2}, 0, 0)$



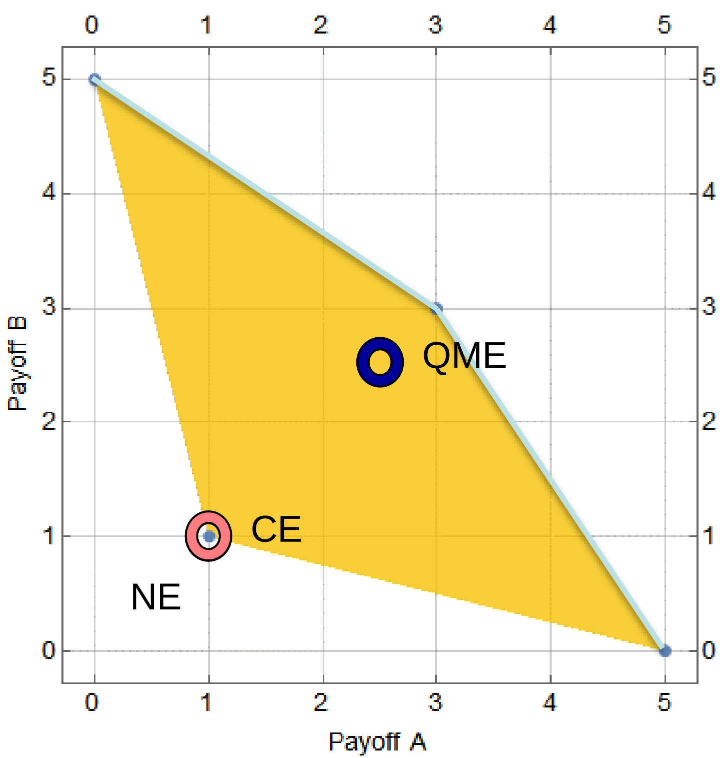


Szopa, M. Efficiency of Classical and Quantum Games Equilibria. Entropy **2021**, 23, 506.

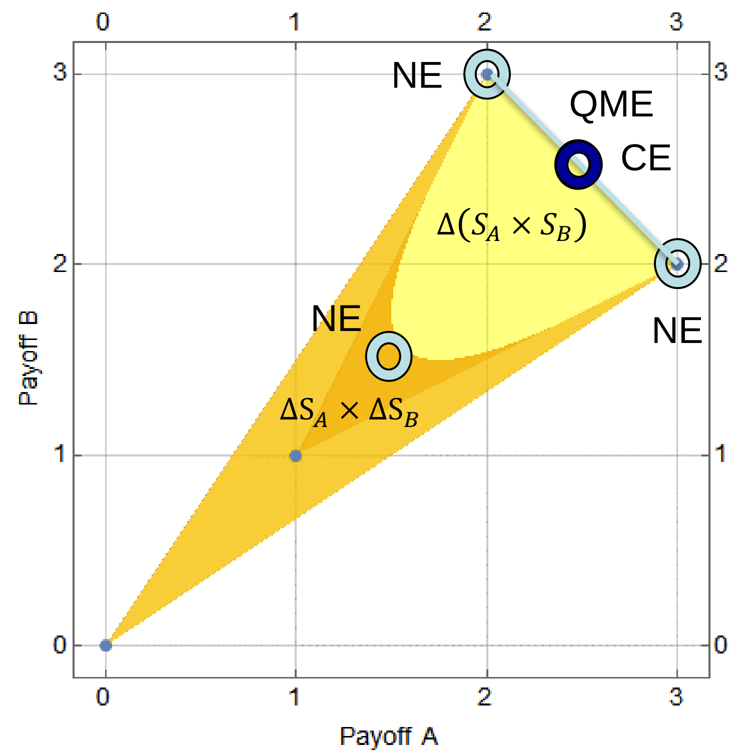
Quantum Mediated Equilibria

		Player B			
		$\widehat{P_0}$	$\widehat{P_x}$	$\widehat{P_y}$	$\widehat{P_z}$
Player A	$\widehat{P_0}$	(v^A_{00}, v^B_{00})	(v^A_{01}, v^B_{01})	(v^A_{10}, v^B_{10})	(v^A_{11}, v^B_{11})
	$\widehat{P_x}$	(v^A_{10}, v^B_{10})	(v^A_{11}, v^B_{11})	(v^A_{00}, v^B_{00})	(v^A_{01}, v^B_{01})
	$\widehat{P_y}$	(v^A_{01}, v^B_{01})	(v^A_{00}, v^B_{00})	(v^A_{11}, v^B_{11})	(v^A_{10}, v^B_{10})
	$\widehat{P_z}$	(v^A_{11}, v^B_{11})	(v^A_{10}, v^B_{10})	(v^A_{01}, v^B_{01})	(v^A_{00}, v^B_{00})

$$\sigma^A = \left(\frac{1}{2}, 0, 0, \frac{1}{2}\right), \sigma^B = \left(0, \frac{1}{2}, \frac{1}{2}, 0\right)$$



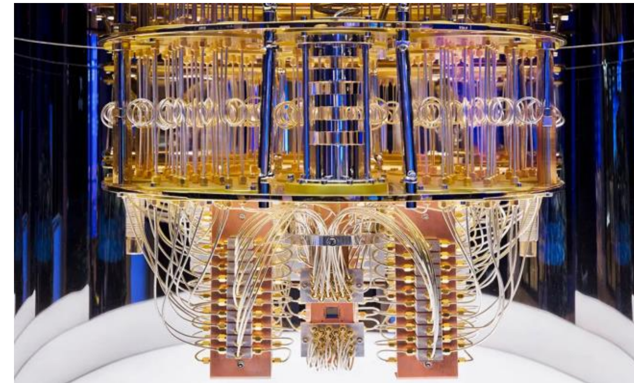
$$\sigma^A = \sigma^B = \left(\frac{1}{2}, 0, 0, \frac{1}{2}\right)$$

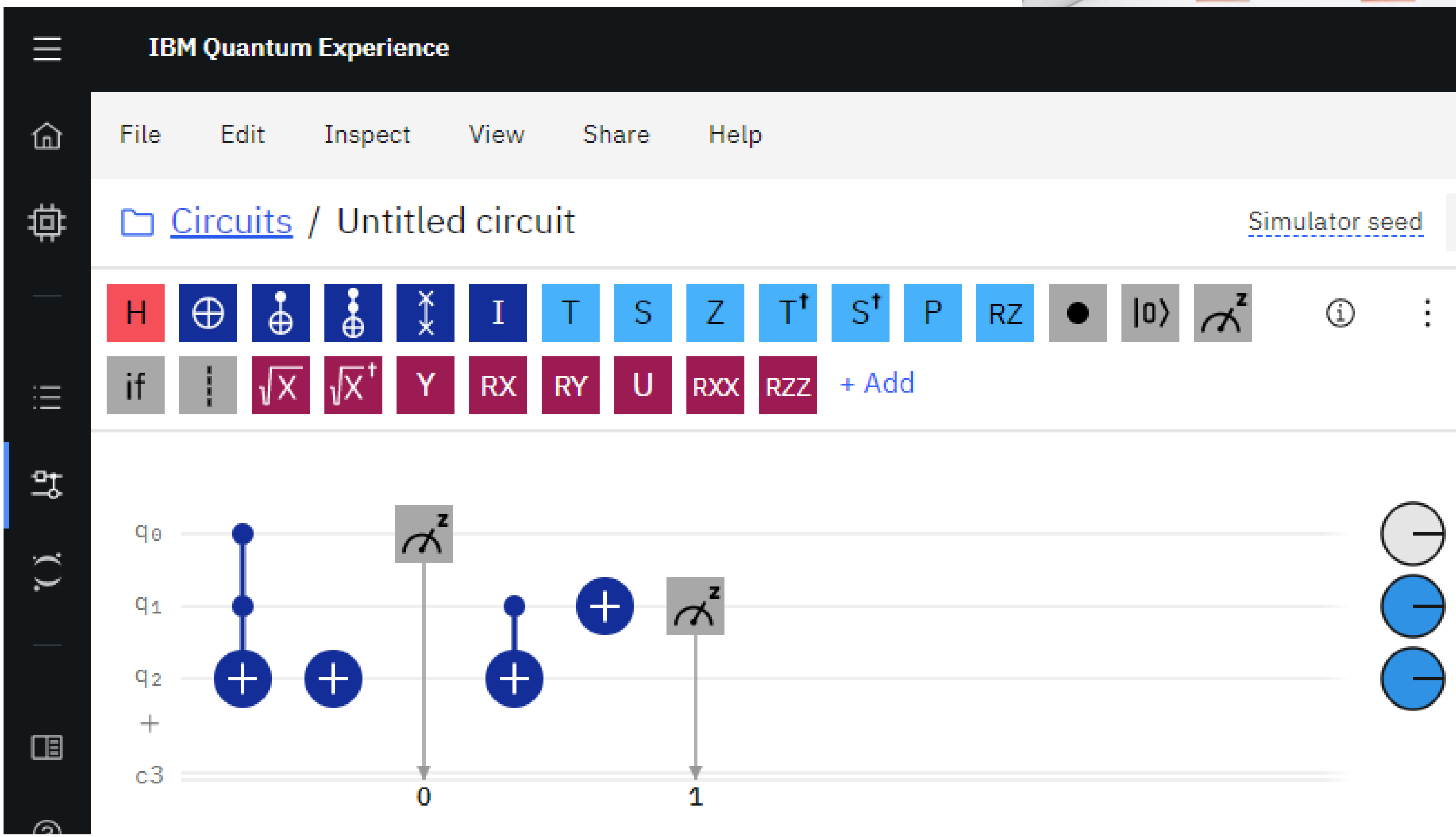


Szopa, M. Efficiency of Classical and Quantum Games Equilibria. Entropy **2021**, 23, 506.

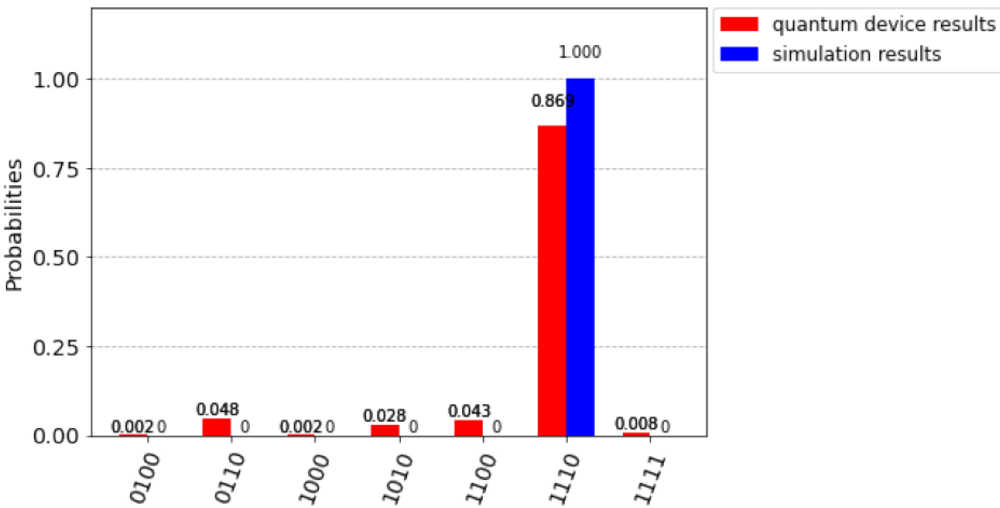
Quantum Computer

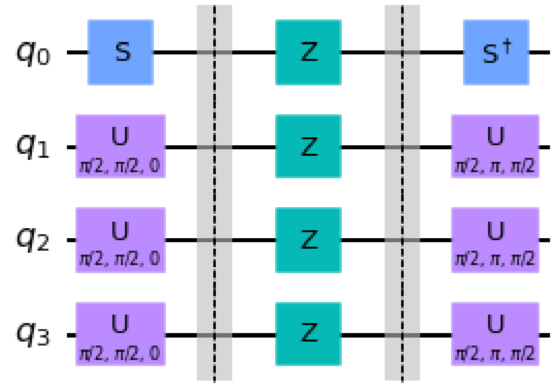
https://quantum-computing.ibm.com/

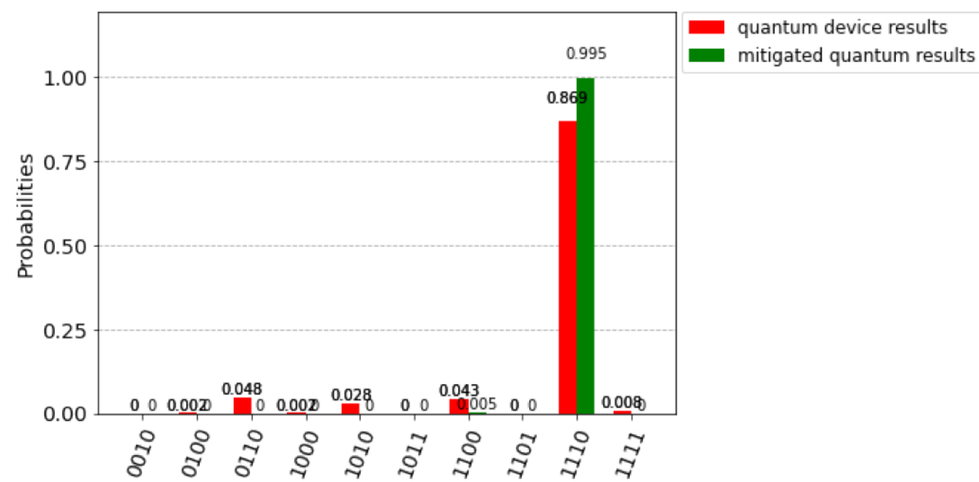




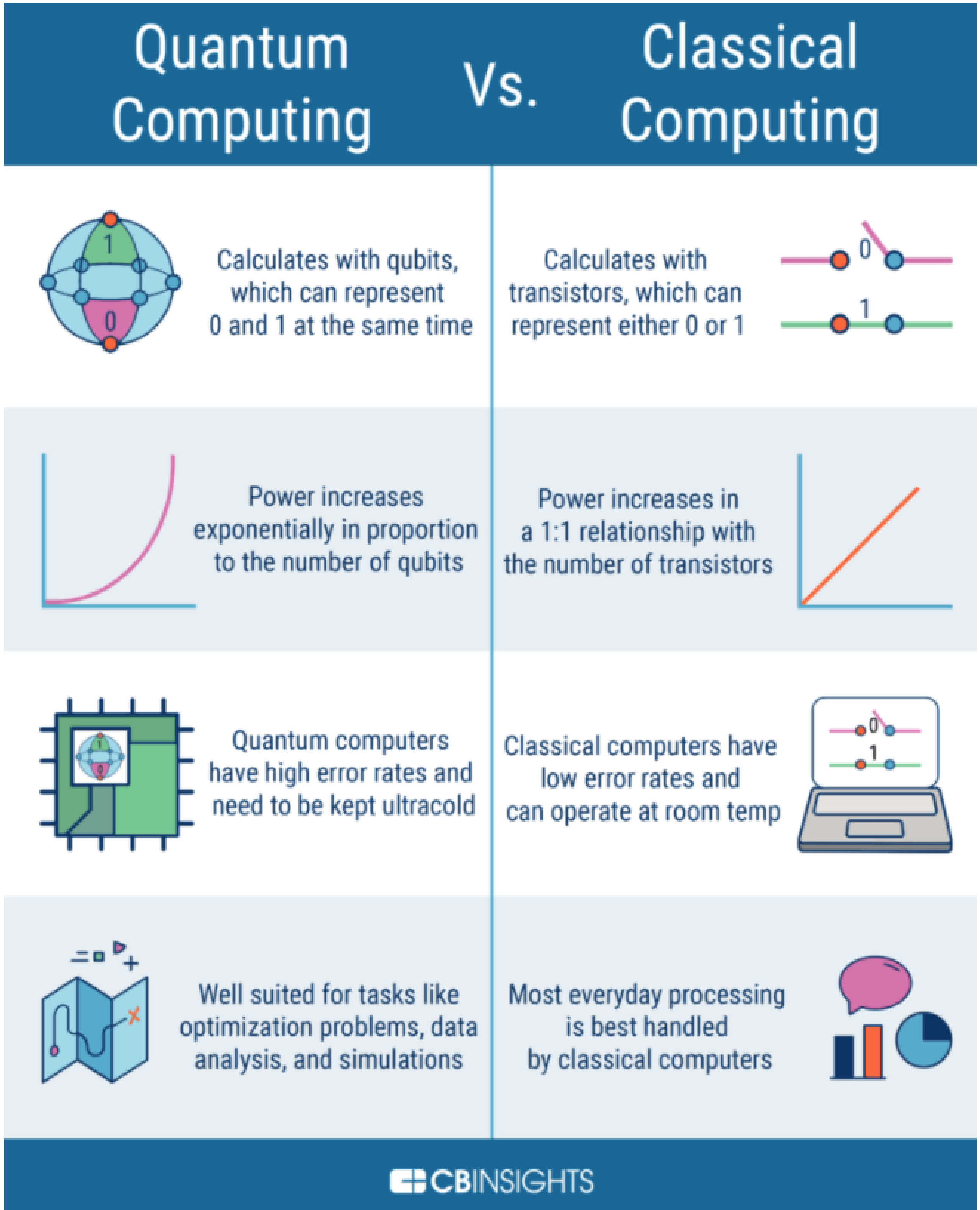
Quantum absentminded driver on IBM-Q







P. Frąckiewicz, K. Rycerz, M. Szopa, *Quantum absentminded driver problem revisited*, subm. to Quantum Inf. Processing


Quantum Computing
Vs.
Classical Computing
1
0
Calculates with qubits, which can represent 0 and 1 at the same time
Calculates with transistors, which can represent either 0 or 1
0
1
Power increases exponentially in proportion to the number of qubits
Power increases in a 1:1 relationship with the number of transistors
Quantum computers have high error rates and need to be kept ultracold
Classical computers have low error rates and can operate at room temp
0
1
Well suited for tasks like optimization problems, data analysis, and simulations
Most everyday processing is best handled by classical computers
CBINSIGHTS

Wnioski

1. Skorelowane równowagi znacznie poprawiają paretoefektywność równowag Nasha ale wymagają urządzenia korelującego, które może być zmanipulowane
2. Gry kwantowe umożliwiają stosowanie strategii niedostępne dla gier klasycznych
3. Równowagi Nasha gier kwantowych są bliskie paretoefektywności równowag skorelowanych
4. Gry kwantowe uniemożliwiają manipulowanie wynikami