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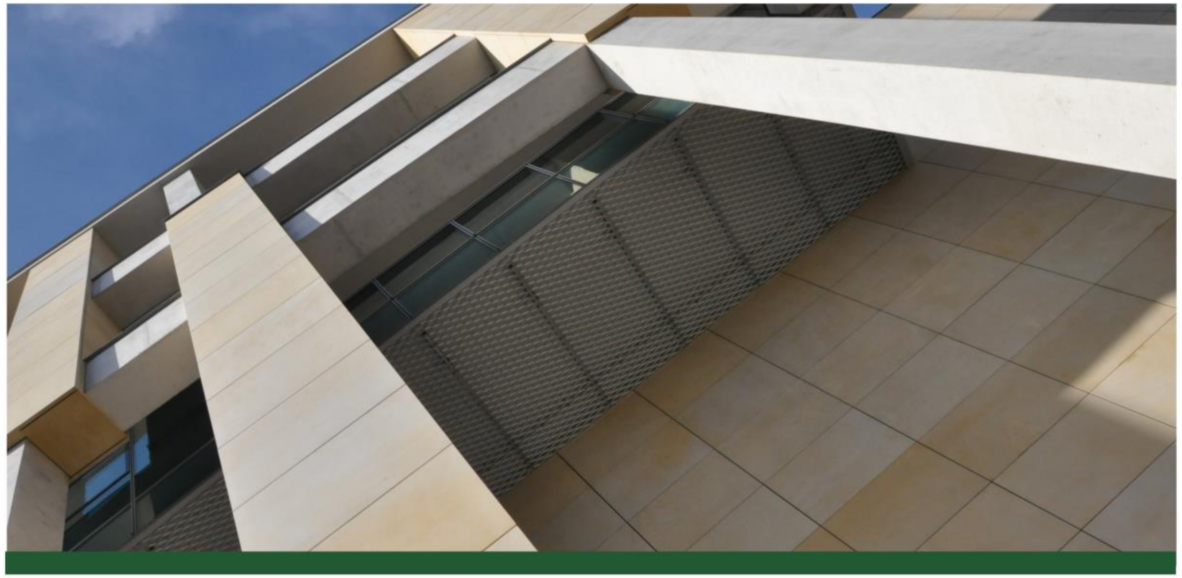

Different ways of extending order scales dedicated to credit risk assessment

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Fuzzy numbers

Fuzzy number (FN) is usually defined as a fuzzy subset of the real line $\mathbb{R}$. The most general definition of FN was formulated by Dubois and Prade. The set of all FN is denoted by the symbol $\mathbb{F}$.

The notion of ordered FN was introduced by Kosiński et al. From formal reasons, the Kosiński's theory had to be revised. In revised theory, the notion of ordered FN is narrowed down to the notion of oriented FN (OFN). On the other hand, arithmetic operations determined for any OFN have a very high level of complexity. For this reason, we restrict our considerations to the case of trapezoidal OFNs (TrOFN).

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Arithmetics of TrOFNs

Definition 1. For any monotonic sequence $(a,b,c,d) \subset \mathbb{R}$, TrOFN $\overleftrightarrow{Tr}(a,b,c,d) = \overleftrightarrow{\mathcal{T}}$ is the pair of orientation $\overrightarrow{a,d} = (a,d)$ and FN $\mathcal{T} \in \mathbb{F}$ described by membership function $\mu_T(\cdot\,|a,b,c,d) \in [0,1]^{\mathbb{R}}$ given by the identity

$$\mu_T(x) = \mu_{Tr}(x|a,b,c,d) = \begin{cases} 0, & x \notin [a,d] \equiv [d,a], \\ \dfrac{x-a}{b-a}, & x \in [a,b[\equiv]a,b], \\ 1, & x \in [b,c] \equiv [c,b], \\ \dfrac{x-d}{c-d}, & x \in]c,d] \equiv [c,d[. \end{cases}$$

If $a < d$ then TrOFN $\overleftrightarrow{Tr}(a,b,c,d)$ has the positive orientation $\overrightarrow{a,d}$ which informs us about possibility of an increase in approximated number. The space of all positively oriented TrOFNs is denoted by the symbol $\mathbb{K}_{Tr}^{+}$.

If $a > d$, then OFN $\overleftrightarrow{Tr}(a,b,c,d)$ has the negative orientation $\overrightarrow{a,d}$ which informs us about possibility of a decrease in approximated number. The space of all negatively oriented TrOFNs we denote by the symbol $\mathbb{K}_{Tr}^{-}$.

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Arithmetics of TrOFNs

For any pair $\left(\overleftrightarrow{Tr}(a,b,c,d), \overleftrightarrow{Tr}(p-a,q-b,r-c,s-d)\right) \in \mathbb{K}_{Tr}^2$ and $\beta \in \mathbb{R}$, arithmetic operations of extended sum $\boxplus$ and dot product $\boxdot$ are defined as follows:

$$\overleftrightarrow{Tr}(a,b,c,d) \boxplus \overleftrightarrow{Tr}(p-a,q-b,r-c,s-d) =$$

$$= \begin{cases} \overrightarrow{Tr}(\min\{p,q\},q,r,\max\{r,s\}), & (q<r) \vee (q=r \wedge p \le s), \\ \overleftarrow{Tr}(\max\{p,q\},q,r,\min\{r,s\}), & (q>r) \vee (q=r \wedge p>s). \end{cases}$$

$$\beta \boxdot \overrightarrow{Tr}(a,b,c,d) = \overrightarrow{Tr}(\beta \cdot a, \beta \cdot b, \beta \cdot c, \beta \cdot d)$$

In general, the TrOFNs addition is not associative [7]. Moreover, for any pair $\left(\overleftrightarrow{Tr}(a,b,c,d), \overleftrightarrow{Tr}(e,f,g,h)\right) \in (\mathbb{K}_{Tr}^{+} \cup \mathbb{R})^2 \cup (\mathbb{K}_{Tr}^{-} \cup \mathbb{R})^2$ we have

$$\overleftrightarrow{Tr}(a,b,c,d) \boxplus \overleftrightarrow{Tr}(e,f,g,h) = \overleftrightarrow{Tr}(a+e,b+f,c+g,d+h)$$

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Order scale dedicated to credit risk assessment

The starting point to determine any order scale is to define Tentative Order Scale (TOS) with the use of linguistic variables. TOS is defined as a following sequence:

$$\overline{TOS} = (X_i)_{i=1}^{n}$$

$$\text{TOS} = \{Bad, Average, Good\}$$

of linguistic labels X_i. Ordering the linguistic labels is then determined by ordering the sequence $\overline{TOS}$. Any TOS can also be enhanced by the intermediate values, which are obtained with the use of perception indicators (PI) given as the sequence

$$\overline{PI} = \left(Y_j\right)_{j=-m}^{j=m}$$

$$\text{PI} = \{much\ below, below, around, above, much\ above\}$$

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Complete order scale (COS)

Cartesian product of sets $\overline{TOS}$ and $\overline{PI}$ forms Extended Order Scale (EOS) determined as the lexicographically ordered set

$$\overline{EOS} = \overline{TOS} \times \overline{PI} = \left\{ \left(X_i, Y_j \right); i = \overline{1,n}, j = \overline{-m,m} \right\} =$$
$$= \left\{ Z_{(2\cdot m+1)\cdot(i-1)+m+1+j}; i = \overline{1,n}, j = \overline{-m,m} \right\} = \left(Z_k \right)_{k=1}^{n\cdot(2\cdot m+1)}$$

TOS and EOS might also be characterised by Numerical Order Scale (NOS)

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Complete order scale (COS)

TOS	EOS	Semantic Meaning	NOS
	$C--$	much below Bad	$\overleftrightarrow{Tr}\left(1,1,\frac{3}{4},\frac{1}{4}\right)$
	$C-$	below Bad	$\overleftrightarrow{Tr}\left(\frac{5}{4},1,\frac{3}{4},\frac{2}{4}\right)$
	$C\sim$	around Bad	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$
C		Bad	$\overleftrightarrow{Tr}(1,1,1,1)$
	$C+$	above Bad	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$
	$C++$	much above Bad	$\overleftrightarrow{Tr}\left(1,1,\frac{5}{4},\frac{7}{4}\right)$
	$B--$	much below Average	$\overleftrightarrow{Tr}\left(2,2,\frac{7}{4},\frac{5}{4}\right)$
	$B-$	below Average	$\overleftrightarrow{Tr}\left(\frac{9}{4},2,\frac{7}{4},\frac{6}{4}\right)$
	$B\sim$	around Average	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
B		Average	$\overleftrightarrow{Tr}(2,2,2,2)$
	$B+$	above Average	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$
	$B++$	much above Average	$\overleftrightarrow{Tr}\left(2,2,\frac{9}{4},\frac{11}{4}\right)$
	$A--$	much below Good	$\overleftrightarrow{Tr}\left(3,3,\frac{11}{4},\frac{9}{4}\right)$
	$A-$	below Good	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$
	$A\sim$	around Good	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
A		Good	$\overleftrightarrow{Tr}(3,3,3,3)$
	$A+$	above Good	$\overleftrightarrow{Tr}\left(\frac{11}{4},3,\frac{13}{4},\frac{14}{4}\right)$
	$A++$	much above Good	$\overleftrightarrow{Tr}\left(3,3,\frac{13}{4},\frac{15}{4}\right)$

Scoring function for borrowers' assessment

Each credit application $\mathcal{A}$ is evaluated by the experts from the point of view of a criteria set $\Phi = \{\mathcal{C}_l : l = 1,2,\dots,p\}$.

The outcome of this assessment is to attribute each credit application $\mathcal{A}$ with the set of partial assessments

$$\Psi(\mathcal{A}) = \left\{\overleftrightarrow{Tr}(\mathcal{A}, C_l) = \overleftrightarrow{Tr}(a_l, b_l, c_l, d_l) : l = 1,2,\dots,p\right\}$$

partial sum of scoring function

$$\overleftrightarrow{\mathcal{S}}^{+}(\mathcal{A}) = \boxplus_{\mathcal{C}_l \in \Phi^{+}(\mathcal{A})} \overleftrightarrow{Tr}(\mathcal{A}, C_l) = \overline{Tr}\left(\sum_{\mathcal{C}_1 \in \Phi^{+}(\mathcal{A})} a_l, \sum_{\mathcal{C}_l \in \Phi^{+}(\mathcal{A})} b_l, \sum_{\mathcal{C}_l \in \Phi^{+}(\mathcal{A})} c_l, \sum_{\mathcal{C}_l \in \Phi^{+}(\mathcal{A})} d_l\right)$$

$$\overleftrightarrow{\mathcal{S}}^{-}(\mathcal{A}) = \boxplus_{\mathcal{C}_l \in \Phi^{-}(\mathcal{A})} \overleftrightarrow{Tr}(\mathcal{A}, C_l) = \overline{Tr}\left(\sum_{\mathcal{C}_1 \in \Phi^{-}(\mathcal{A})} a_l, \sum_{\mathcal{C}_l \in \Phi^{-}(\mathcal{A})} b_l, \sum_{\mathcal{C}_l \in \Phi^{-}(\mathcal{A})} c_l, \sum_{\mathcal{C}_l \in \Phi^{-}(\mathcal{A})} d_l\right)$$

Finally, we calculate the value $\overleftrightarrow{\mathcal{S}}(\mathcal{A})$ of scoring function

$$\overleftrightarrow{\mathcal{S}}(\mathcal{A}) = p^{-1} \boxdot \left(\overleftrightarrow{\mathcal{S}}^{+}(\mathcal{A}) \boxplus \overleftrightarrow{\mathcal{S}}^{-}(\mathcal{A})\right).$$

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Simplification of Complete Order Scale (1)

Simplified forms of COS:

- One-stage COS2 using the PI set {much below, about, much above}
- One-stage COS3 using the PI set {below, about, above}
- Zero-stage COS4 using the PI set {about}

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Simplification of Complete Order Scale (2)

- COS2 is derived from COS1 by replacing PI {*below, above*} respectively by PI {*much below, much above*}
- COS3 is derived from COS1 by replacing PI {*much below, much above*} respectively by PI {*below, above*}
- COS4 is derived from COS1 by replacing PI {*much below, below, around, above, much above*} by the PI {a*bove*}

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Example

No.	Criteria	COS1 EOS	COS1 NOS	COS2 EOS	COS2 NOS	COS3 EOS	COS3 NOS	COS4 EOS	COS4 NOS
1	prospects of business	C+	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$	C++	$\overleftrightarrow{Tr}\left(1,1,\frac{5}{4},\frac{7}{4}\right)$	C+	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$	C~	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$
2	Board members experience	A++	$\overleftrightarrow{Tr}\left(3,3,\frac{13}{4},\frac{15}{4}\right)$	A++	$\overleftrightarrow{Tr}\left(3,3,\frac{13}{4},\frac{15}{4}\right)$	A+	$\overleftrightarrow{Tr}\left(\frac{11}{4},3,\frac{13}{4},\frac{14}{4}\right)$	A~	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
3	Chairperson experience	A++	$\overleftrightarrow{Tr}\left(3,3,\frac{13}{4},\frac{15}{4}\right)$	A++	$\overleftrightarrow{Tr}\left(3,3,\frac{13}{4},\frac{15}{4}\right)$	A+	$\overleftrightarrow{Tr}\left(\frac{11}{4},3,\frac{13}{4},\frac{14}{4}\right)$	A~	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
4	operations range regional	C+	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$	C++	$\overleftrightarrow{Tr}\left(1,1,\frac{5}{4},\frac{7}{4}\right)$	C+	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$	C~	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$
5	operations range international	A−−	$\overleftrightarrow{Tr}\left(3,3,\frac{11}{4},\frac{9}{4}\right)$	A−−	$\overleftrightarrow{Tr}\left(3,3,\frac{11}{4},\frac{9}{4}\right)$	A−	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A~	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
6	risk associated with market	B++	$\overleftrightarrow{Tr}\left(2,2,\frac{9}{4},\frac{11}{4}\right)$	B++	$\overleftrightarrow{Tr}\left(2,2,\frac{9}{4},\frac{11}{4}\right)$	B+	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$	B~	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
7	risk associated with trade	B+	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$	B++	$\overleftrightarrow{Tr}\left(2,2,\frac{9}{4},\frac{11}{4}\right)$	B+	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$	B~	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
8	risk associated with suppliers	A−	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A−−	$\overleftrightarrow{Tr}\left(3,3,\frac{11}{4},\frac{9}{4}\right)$	A−	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A~	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
9	risk associated with customers	A−	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A−−	$\overleftrightarrow{Tr}\left(3,3,\frac{11}{4},\frac{9}{4}\right)$	A−	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A~	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
10	diversification—products	B~	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$	B~	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$	B~	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$	B~	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
11	diversification—sales markets	C~	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$	C~	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$	C~	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$	C~	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$
12	diversification—supply market	B~	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$	B~	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$	B~	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$	B~	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
	Scoring value		$\overleftrightarrow{Tr}\left(\frac{97}{48},\frac{104}{48},\frac{107}{48},\frac{118}{48}\right)$		$\overleftrightarrow{Tr}\left(\frac{98}{48},\frac{104}{48},\frac{107}{48},\frac{119}{48}\right)$		$\overleftrightarrow{Tr}\left(\frac{95}{48},\frac{104}{48},\frac{107}{48},\frac{116}{48}\right)$		$\overleftrightarrow{Tr}\left(\frac{80}{48},\frac{104}{48},\frac{104}{48},\frac{128}{48}\right)$

Conclusions

- In experts' evaluation there is always a significant degree of imprecision
- Assessments presented by experts can vary even if the linguistic vairable is the same or a close one
- Experts often want a simplification of scales, numbers of degrees etc. as too many levels of given scale are incomprehensible
- The paper presents a formal structure of COS. The knowledge of that structure allows for the transformation of a given two-stage COS to less complex structures.
- It was noticed that the change of COS structure influences the value of a scoring function.
- the study referring to the imprecision of a scoring function should be conducted

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Thank you for your attention

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