Horizontal cooperation in logistics - game theory approach

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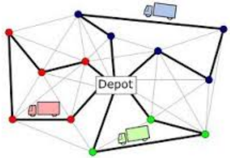
Originality These considerations identify a practical and academic foundation for further research.

Cooperation in supply chains

Setting up a cooperation to improve the own performance is an effective way to improve logistic operations.

Types of problems:

- transportation planning
- traveling salesman
- vehicle routing
- joint distribution



Benefits of collaboration:

- the potential cost savings often range from 5% to 15% (Cruijssen and Salomon; 2004),
- Krajewska and Kopfer (2008) show that cooperation between two carriers yields 10% reduction in the number of vehicule and 12.46% reduction in routing cost,
- Ergun et al. (2007) focuses on minimizing execution costs for a coalition of freight forwarders.

Cruijssen, F. C. A. M., and Salomon, M. (2004). Empirical study: Order sharing between transportation companies may result in cost reductions between 5 to 15 percent.
Ergun, O., Kuyzu, G., and Savelsbergh, M. (2007). Shipper collaboration. Computers and Operations Research, 34(6), 1551-1560.
Krajewska, M. A., Kopfer, H., Laporte, G., Ropke, S., and Zaccour, G. (2008). Horizontal cooperation among freight carriers: request allocation and profit sharing. Journal of the Operational Research Society, 59(11), 1483-1491.

A cooperative game is a pair $G = (N, \nu)$, $N = \{1, 2, ..., n\}$. The characteristic function ν assign to every nonempty coalition $S \subseteq N$ a value $\nu(S)$, with $\nu(\emptyset) = 0$. When coalition S cooperates, the total cost $C(S)$ and the resulting cost saving is given by

$$\nu(S) = \sum_{i \in S} C(\{i\}) - C(S), \quad \text{for all} \quad S \subseteq N.$$

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$\text{Core(C)} = \{x \in R^n : \sum_{j \in N} x_j = C(N), \sum_{j \in S} x_j \leq C(S), \forall S \in K.\}$

Definition

The game $G = (N, \nu)$ is superadditive if the value ν satisfies the following equation

$\nu(S \cup T) \geq \nu(S) + \nu(T)$ for all coalitions $S, T \subseteq N$ and $S \cap T = \emptyset$

Allocation methods - overview

Most problems in collaborative transportation use concepts and methods from cooperative game theory. We have found that more than 40 different cost allocation methods have been used in the literature on collaborative transportation. Several of these methods come from previous work on cooperative game theory, so we refer to them as traditional methods.

Table: Traditional methods

Shapley value
Nucleolus
Equal methods
Gately point

Guajardo, M., and Rönnqvist, M. (2016). A review on cost allocation methods in collaborative transportation. International transactions in operational research, 23(3), 371-392.

Game theory concepts

When coalition S cooperates, the total cost $C(S)$ and the resulting cost saving is given by $\nu(S) = \sum_{i \in S} C(\{i\}) - C(S), \quad \text{for all} \quad S \subseteq N.$

Definition (Shapley value)

$$\phi_i(\nu) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!}(\nu(S \cup \{i\}) - \nu(S)), \quad \text{for all} \quad i \in S.$$

Definition (Nucleolus)

$$nc(N, \nu) = \{x \in X(N, \nu) : \text{there does not exists } y \in X(N, \nu)), \quad e(y) \succ_{lxm} e(x)$$

Game theory concepts

Definition (Equal allocation)

$$\phi_i(\nu) = \frac{\nu(N)}{n}.$$

Definition (Gately point)

$$G_i(\nu) = \frac{\nu(N) - \nu(N \setminus \{i\})}{\sum_{j \in N}(\nu(N) - \nu(N \setminus \{j\}))}\nu(N).$$

Properties of cost allocations

The major problem in cooperative game theory is how to allocate the costs of the grand coalition among the players. The allocation vector satisfies a variety of properties, some of them are listed below.
Coalitional game $G = (N, \nu)$ is

- ✓ *efficient* if $\sum_{i \in N} \phi_i(\nu) = \nu(N)$, this property ensures that the total value of the grand coalition is distributed among the players,

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- ✓ *individual rationality* if $\forall_{i \in N} \phi_i(\nu) \geq \nu(\{i\})$, it guarantees that each player should at least get what the player would get individually,
- ✓ *collective rationality* if $\forall_{i \in S} \phi_i(\nu) \geq \nu(S), \quad \emptyset \subset S \subset N$. If this property is not satisfied, then the players have an incentive to drop out of the grand coalition in order to gain a higher payoff allocation.

The payoffs that secures efficiency and individual rationality are called the *imputation*. No partner would accept an allocation that has no imputation, therefore most of the solution concepts are a set of imputations.

Let us consider the example of a group of retailers who decide to cooperate by ordering jointly via a single large order. The mathematical model is developed based on the following assumptions

- the group of retailers can only make orders for full truckload delivery,
- the demand of each retailer is deterministic, and there is no shortage,
- the transportation cost is not relevant to the transportation quantity.

In the optimal replenishment strategy under full truckload (FTL) shipments ($V_i \cdot Q_i = CAP_i$), the carriers have a similar cost structure, and

$$F_S = \sum_{i \in S} \frac{D_i \cdot V_i}{Cap_i} = \sum_{i \in S} F_i, \quad \emptyset \subset S \subseteq N, \ i \in \{1, 2, ..., n\}.$$

Q_i	order size of i retailer	Cap_i	vehicle capacity
V_i	volume of i retailer' product	F_i	ordering frequency of retailer i
D_i	demand of retailer i		

Qu, H., Wang, L., Liu, R., 2015. A contrastive study of the stochastic location-inventory problem with joint replenishment and independent replenishment. Expert Systems with Applications 42.

$C(S)$

The total average cost of the coalition $\emptyset \subset S \subseteq N$ is the sum of the ordering cost, holding cost and transportation cost, as follows

$$\begin{aligned} C(S) &= \underbrace{A \cdot \sum_{i \in S} F_i}_{\text{ordering cost}} + \underbrace{\frac{\sum_{i \in S} H_i}{\sum_{i \in S} F_i}}_{\text{holding cost}} + \underbrace{\sum_{i \in S} \beta_i Dist_i}_{\text{transportation cost}} \\ &= CO(S) + CH(S) + CT(S), \quad i \in \{1, 2, ..., n\}. \end{aligned}$$

$Dist_i$	travel distance		
β_i	cost per kilometer	$CO(S)$	ordering cost of coalition S
H_i	holding cost	$CH(S)$	holding cost of coalition S
F_i	ordering frequency of retailer i	$CT(S)$	travel cost of coalition S
A	fixed ordering cost	$C(S)$	total cost of coalition S

An illustrative example of four retailers (label A, B, C, D) supplied by the same manufacturer is presented and analysed. The travel distance, the storage and ordering cost, product volume, vehicle capacity, demand of each retailer, and ordering frequency are shown in table below.

$Dist_i$	H_i	A	V_i	Cap_i	D_i	F_i	$CO(S)$	$CH(S)$	$CT(S)$	$C(S)$
150	300	20	1	50	600	12	240	25	600	865
170	100	20	1	50	100	2	40	50	680	770
150	100	20	1	50	50	1	20	100	600	720
130	200	20	1	50	150	5	100	45	500	645

$Dist_i$	travel distance	Cap_i	vehicle capacity
D_i	demand of retailer i	$CO(S)$	ordering cost of coalition S
H_i	holding cost	$CH(S)$	holding cost of coalition S
F_i	ordering frequency of retailer i	$CT(S)$	travel cost of coalition S
A	fixed ordering cost	$C(S)$	total cost of coalition S
V_i	volume of i retailer's product		

Coalition	$C(S)$	$CS(S)$
$\{A\}$	865.0	0.0
$\{B\}$	770.0	0.0
$\{C\}$	720.0	0.0
$\{D\}$	645.0	0.0
$\{A, B\}$	1 204.6	430.4
$\{A, C\}$	1 130.8	454.2
$\{A, D\}$	1 153.4	356.6
$\{B, C\}$	1 022.7	467.3
$\{B, D\}$	1 022.9	393.1
$\{C, D\}$	954.0	411.0
$\{A, B, C\}$	1 649.3	705.7
$\{A, B, D\}$	1 671.6	608.4
$\{A, C, D\}$	1 597.3	632.7
$\{B, C, D\}$	1 470.0	665.0
$\{A, B, C, D\}$	2 115.0	885.0

The grand coalition $\{A, B, C, D\}$ is stable, which means that no subcoalition has an incentive to leave the grand coalition. In addition, for the CS function, the monotonicity and superadditivity properties hold.

Carriers in coalitions	A	B	C	D
Stand-alone cost	865	770	720	645
	Equal allocation			
Cost saving	221.3	221.3	221.3	221.3
	Shapley value			
Cost saving	214.8	233.7	248.9	187.7
	Nucleolus			
Cost saving	209.2	241.5	265.8	168.5
	Gately point			
Cost saving	209.8	240.6	263.7	171.0

	Equal allocation			
Cost saving	221.3	221.3	221.3	221.3
Net cost	643.8	548.8	498.8	423.8
Savings ratio	25.6 %	28.7 %	30.7 %	34.3 %
	Shapley value			
Cost saving	214.8	233.7	248.9	187.7
Net cost	650.2	536.3	471.1	457.3
Savings ratio	24.8 %	30.3 %	34.6 %	29.1 %
	Nucleolus			
Cost saving	209.2	241.5	265.8	168.5
Net cost	655.8	528.5	454.2	476.5
Savings ratio	24.2 %	31.4 %	36.9 %	26.1 %
	Gately point			
Cost saving	209.8	240.6	263.7	171.0
Net cost	655.2	529.4	456.3	474.0
Savings ratio	24.2 %	31.2 %	36.6 %	26.5 %

Net Cost equals the Stand-alone Cost minus Cost Savings. The Savings Ratio equals the Cost Savings divided by the Net Cost.

Further research may take a number of different directions.

- One such direction is to consider multiple suppliers
- Another possible direction is to extend the model by considering multiple objectives, such as the supply chain risk.
- Finally, an assumption of the approach discussed in the paper is that all the trips are full-truck-load.
- The long-term goal is to elaborate the paper to consider less-than-truck load scenarios.

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Thank you for your attention

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